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Minimal String Unification and Yukawa Couplings in Orbifold Models

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Abstract

We study the minimal supersymmetric standard model derived from $Z_N \times Z_M$ orbifold models. Moduli dependent threshold corrections of the gauge couplings are investigated to explain the measured values of the coupling constants. Also we study Yukawa couplings of the models. We find that the $Z_2 \times Z'_6$, $Z_2 \times Z_6$, $Z_3 \times Z_6$ and $Z_6 \times Z_6$ orbifold models have the possibility to derive Yukawa couplings for the second and third generations as well as the measured gauge coupling constants. Allowed models are shown explicitly by combinations of modular weights for the matter fields.

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1. Introduction

Superstring theory is the best candidate for a unified theory of all the known interactions including gravity. All the gauge coupling constants coincide even without a grand unified group at a string scale $M_{\text{st}} = 5.27 \times g_{\text{st}} \times 10^{17} \text{GeV}$ [1], where $g_{\text{st}} \simeq 1/\sqrt{2}$ is a universal string coupling. Yukawa couplings have the same origin as the gauge coupling constants.

A minimal string model is the 4-dim string vacuum which has the same massless spectrum as the minimal supersymmetric standard model (MSSM). That is one of the simplest scenarios to lead from the string theories to the low energy physics. The minimal string models do not face the problems of the fast proton decay and triplet-doublet splitting in the Higgs sector as well as mass splitting between the quarks and the leptons other than the third generation, unlike the GUTs or supersymmetric standard models with extra matter fields. However the minimal string models are not consistent with recent study of the LEP measurements which show all the gauge coupling constants unify simultaneously at $M_X \simeq 10^{16} \text{GeV}$ within the framework of the MSSM [2]. We need some threshold corrections at M_{st} in order to explain the difference between M_{st} and M_X .

An orbifold construction is one of the simplest and most interesting methods to construct 4-dim string vacua [3, 4]. Threshold corrections to gauge couplings of the orbifold models were studied in ref.[5, 6]. The correction depends on a moduli parameter T , which describes a geometrical feature of the orbifold like its size. The parameter is obtained as a vacuum expectation value of a moduli field, when the supersymmetry (SUSY) breaks. Refs.[7, 8] show that the moduli field take the vacuum expectation value of order one in the SUSY-breaking due to a gaugino condensation. Other phenomenological aspects have been studied like Yukawa couplings, Kähler potential and so on. Further the minimal string models have been searched explicitly [9].

Using the threshold corrections, recent work [10-14] showed the possibility to derive the gauge coupling constants consistent with the measurements within the framework of the minimal string models from the orbifold models. Ref.[11] showed that Z_6 -II, Z_8 -I and $Z_N \times Z_M$ orbifold models are promising in the case where a level k_1 of $U(1)_Y$ is equal to $5/3$. However string theories obtain any other values of k_1 [15]. Refs.[12, 14] investigated the case with the general values

of k_1 and showed the Z_6 -I orbifold models as well as the above Z_N are promising, and the $Z_2 \times Z_2$ and $Z_3 \times Z_3$ orbifold models are ruled out in the case where an overall moduli parameter is of order one and the SUSY breaks at M_Z . Further it is shown that the $Z_3 \times Z_3$ orbifold models are allowed in the case where the SUSY-breaking scale is 1TeV. Thus this constraint to derive the measured gauge coupling constants at M_Z is very useful to select models from the huge number of 4-dim string vacua for the Z_N orbifold models, while the $Z_N \times Z_M$ orbifold models are not constrained so much. Actually explicit information on the allowed Z_N orbifold models is shown in ref.[14].

It is very important to extend the above analyses to the prediction of the Yukawa coupling values for the quarks and the leptons. Selection rules for the couplings are very restricted in the orbifold models [16-19]. Thus, before the prediction of the Yukawa coupling values at M_Z it is useful to study which models allow realistic Yukawa couplings among the minimal string models obtained at the above stage. Ref.[14] discussed Yukawa couplings of some models derived from the Z_N orbifold constructions so as to show the minimal string models can allow only the Yukawa couplings for the top and bottom quarks as renormalizable couplings. The other couplings might be explained due to nonrenormalizable couplings. Here following the approach of ref.[20], we assume that the second and third generations of the quarks and the leptons have renormalizable couplings and the couplings for the first generation could be explained by nonrenormalizable couplings. In this paper we study the $Z_N \times Z_M$ models to allow the above type of the Yukawa couplings among the minimal string models which have the gauge coupling constants consistent with the measured values at M_Z through the threshold corrections. We discuss mainly the cases with T of order one. Further we show explicitly the models. That is very useful for model building.

This paper is organized as follows. In section two we review briefly the $Z_N \times Z_M$ orbifold models. Then we study which sector are possible to have each MSSM matter field under a certain value of k_1 . Larger values of k_1 are required so that there are the matter fields in the oscillated states. In section three the threshold corrections of the orbifold models are reviewed. We show which modular weights are allowed for the MSSM matter fields under a certain value of k_1 . In section four we study the possibility to derive minimal string models consistent with the measurements of the gauge coupling constants. In section five we investigate the Yukawa couplings allowed in the minimal string models obtained in section

four. That constrains quite the promising models. We show explicitly the results. Section six is devoted to conclusions and discussions.

2. $Z_N \times Z_M$ Orbifold Models

In this section we review the $Z_N \times Z_M$ orbifold models [4]. In the orbifold construction, the string states consist of left-moving and right-moving bosonic strings on the 4-dim space-time and a 6-dim $Z_N \times Z_M$ orbifold, a left-moving gauge part and a right-moving fermionic string which is related to the right-moving bosonic string through a world-sheet SUSY. We bosonize the fermionic string so as to obtain the bosonic string whose momenta span an $SO(10)$ lattice. Momenta of the gauge parts span an $E_8 \times E'_8$ lattice. The $Z_N \times Z_M$ orbifolds are obtained by dividing tori in terms of two independent twists θ and ω , where $\theta^N = \omega^M = 1$. We denote eigenvalues of θ and ω in a complex basis (X_i, \tilde{X}_i) ($i = 1, 2, 3$) as $\exp[2\pi i v_1^i]$ and $\exp[2\pi i v_2^i]$, respectively. The exponents for each $Z_N \times Z_M$ orbifold model are shown in the second column of Table 1. When the 6-dim torus is divided by θ and ω , the $SO(10)$ and $E_8 \times E'_8$ lattices are simultaneously divided by some shifts. Then we obtain only $N = 1$ space-time SUSY and a smaller gauge group. Further the $E_8 \times E'_8$ lattice is shifted by Wilson lines [21, 22, 19]. Here we assume that through the above procedure we obtain the $SU(3) \times SU(2) \times U(1)_Y$ gauge group in the observable sector.

Closed strings on the orbifolds are classified into two types. One is an untwisted string and the other is a twisted string. The former closes even on the torus and has the following massless condition for the left-mover,

$$h - 1 = 0, \quad (2.1)$$

where h is a conformal dimension due to the $E_8 \times E'_8$ gauge part. The twisted string has a boundary condition twisted by $\theta^\ell \omega^m$. Massless states of the $\theta^\ell \omega^m$ -twisted sector $T_{\ell m}$ should satisfy the following condition,

$$h + N_{OSC} + c_{\ell m} - 1 = 0, \quad (2.2)$$

where N_{OSC} is the oscillator number. Here $c_{\ell m}$ is the ground state energy obtained as

$$c_{\ell m} = \frac{1}{2} \sum_{i=1}^3 v_{\ell m}^i (1 - v_{\ell m}^i),$$

$$v_{\ell m}^i \equiv \ell v_1^i + m v_2^i - \text{Int}(\ell v_1^i + m v_2^i), \quad (2.3)$$

where $\text{Int}(a)$ represents an integer part of a .

A state with an \underline{R} representation under a non-abelian group G contributes to the conformal dimension as follows,

$$h = \frac{C(\underline{R})}{C(G) + k}, \quad (2.4)$$

where k is a level of a Kac-Moody algebra corresponding to G and $C(\underline{R})$ ($C(G)$) denotes a quadratic Casimir of the \underline{R} (adjoint) representation. For example we obtain $C(\text{SU}(N)) = N$ for the adjoint representation of $\text{SU}(N)$. The orbifold models in general lead to $k = 1$ for the non-abelian group, although we can obtain the models with higher levels by a complicated construction [23]. Therefore we restrict ourselves to the case with $k = 1$ for the non-abelian groups. On the other hand, the state with a charge Y of the $\text{U}(1)_Y$ has another contribution to the conformal dimension as Y^2/k_1 , where k_1 is the level for the $\text{U}(1)_Y$. The value of k_1 is the free parameter in the minimal string models. Further the matter fields have charges under extra $\text{U}(1)$ s, which might be broken, and the extra charges contribute to the conformal dimension.

Using the above discussion, we have constraints on massless spectra of the MSSM matter fields. For example we consider the quark doublets with $Y = 1/6$. The representation $(3,2)$ under the $\text{SU}(3) \times \text{SU}(2)$ has a contribution to the conformal dimension by $7/12$ through (2.4). For the quark doublets to have N_{OSC} in $T_{\ell m}$, the level k_1 should satisfy the following relation,

$$k_1 \geq \frac{1}{36(5/12 - c_{\ell m} - N_{OSC})}. \quad (2.5)$$

The case with $c_{\ell m} = N_{OSC} = 0$ corresponds to the untwisted sector. Thus the level k_1 should satisfy $k_1 > 1/15$ so that the quark doublets appear in the untwisted sector. Similarly we obtain condition that the quark doublets and the other MSSM matter fields exist in each sector and have the oscillator number N_{OSC} . Existence of the lepton singlets with $Y = 1$ derives the lower bound of k_1 as $k_1 \geq 1$.

3. Duality Symmetry and Threshold Corrections

Spectra in the orbifold models are invariant under the following duality transformation [24],

$$T_i \rightarrow \frac{a_i T_i - i b_i}{i c_i T_i + d_i}, \quad (3.1)$$

$$a_i, b_i, c_i, d_i \in \mathbf{Z}, \quad a_i d_i - b_i c_i = 1,$$

where T_i is a moduli parameter describing a geometrical feature of the i -th plane. In this paper we restrict ourselves to the case of an overall moduli parameter, i.e., $T = T_1 = T_2 = T_3$.

Effective theories derived from the orbifold models also have the duality symmetry [25]. In the theories, the moduli field T have the following Kähler potential,

$$-3 \log |T + \bar{T}|. \quad (3.2)$$

A vacuum expectation value of the moduli field gives the geometry of the orbifold. The Kähler potential (3.2) is invariant under the duality symmetry (3.1) up to the Kähler transformation. On the other hand, the Kähler potential of the chiral matter field A is obtained as

$$(T + \bar{T})^n A \bar{A}, \quad (3.3)$$

where n is called modular weight [26, 11]. The untwisted sector has the modular weight $n = -1$. The twisted sector with an unrotated plane has the modular weight $n = -1$, while the twisted sector without unrotated planes has the modular weight $n = -2$. Further the oscillator ∂X_i reduces the modular weight by one and $\partial \tilde{X}_i$ contributes to the modular weight oppositely. The duality invariance of (3.3) requires the following duality transformation of the matter fields,

$$A \rightarrow A(icT + d)^n. \quad (3.4)$$

Using the discussion in the previous section, we have the possible modular weights for the MSSM matter fields under k_1 in each twisted sector of all orbifold models. Table 2 and 3 show lower bounds of k_1 so that the MSSM matter fields have each modular weight. In the tables Q , U , D , L , E and H denote the quark doublets, quark singlets of the up-sector and the down-sector, lepton doublets, lepton singlets and Higgs fields of the MSSM, respectively.

The duality symmetry becomes anomalous in terms of loop effects due to only massless fermions [27, 6]. One-loop effective Lagrangian including the duality

anomalous term is obtained as

$$\mathcal{L}_{\text{nl}} = \sum_a \int d^2\theta \frac{1}{4} W^a W^a [S - \frac{1}{16\pi^2} \frac{1}{16} \square^{-1} \overline{\mathcal{D}} \mathcal{D} \mathcal{D} b'_a \log(T + \bar{T})] + \text{h.c.}, \quad (3.5)$$

where S is a dilaton/axion field, W^a is a Yang-Milles superfield and a is an index for a gauge group. The second term of (3.5) is anomalous under the duality symmetry. Here the duality anomaly coefficients b'_a are obtained as

$$b'_a = -3C(G_a) + \sum_R T(R)(3 + n_R), \quad (3.6)$$

where $T(R)$ is the Dynkin index for the R representation, i.e., $T(R) = 1/2$ for the N -dim fundamental representation of the $SU(N)$.

The duality anomaly can be cancelled by two ways [27, 6]. One is the Green-Schwarz (GS) mechanism [28], which induces the non-trivial transformation to the dilaton field S as follows,

$$S \rightarrow S - \frac{1}{8\pi^2} \delta_{GS} \log(icT + d), \quad (3.7)$$

where δ_{GS} is an unknown GS coefficient. It is remarkable that the GS mechanism is independent of the gauge groups. Further, the duality anomaly can also be cancelled through the moduli dependent threshold corrections $\Delta(T)$ due to the towers of massive modes. The correction is obtained as

$$\Delta_a(T) = -\frac{1}{16\pi^2} (b'_a - k_a \delta_{GS}) \log|\eta(T)|^4, \quad (3.8)$$

where $\eta(T)$ is the Dedekind function, i.e., $\eta(T) = e^{-\pi T/12} \prod_{n=1}^{\infty} (1 - e^{-2\pi n T})$.

Including the threshold corrections, we have the running gauge coupling constant $\alpha_a = k_a g_a^2/4\pi$ at μ as follows,

$$\alpha_a^{-1}(\mu) = \alpha_{\text{st}}^{-1} + \frac{1}{4\pi} \frac{b_a}{k_a} \log \frac{M_{\text{st}}^2}{\mu^2} - \frac{1}{4\pi} (\frac{b'_a}{k_a} - \delta_{GS}) \log[(T + \bar{T}) |\eta(T)|^4], \quad (3.9)$$

where α_{st} is the universal string coupling and b_a are $N = 1$ β -function coefficients, i.e., $b_3 = -3$, $b_2 = 1$ and $b_1 = 11$ for $SU(3)$, $SU(2)$ and $U(1)_Y$ in the MSSM.

We study the unification scale M_X of the $SU(3)$ and $SU(2)$ gauge coupling constants, α_3 and α_2 . Note that the gauge coupling constant of $U(1)_Y$, α_1 does not always coincide with the other couplings at M_Z , because we consider the case

where the level k_1 of $U(1)_Y$ is the general value. We obtain the unification scale M_X as follows,

$$\log(M_X^2/\mu^2) = \pi\{\sin^2\theta_W(\mu)\alpha_{\text{em}}^{-1}(\mu) - \alpha_3^{-1}(\mu)\}. \quad (3.10)$$

Eq.(3.10) is available at μ higher than the soft SUSY-breaking scale. We use the measured values of the gauge couplings as $\sin^2\theta_W(M_Z) = 0.2325 \pm .0008$, $\alpha_{\text{em}}^{-1}(M_Z) = 127.9 \pm .1$, $\alpha_3^{-1}(M_Z) = 8.82 \pm .27$ at $M_Z = 91.173 \pm .020$. If the SUSY breaks at M_Z , the gauge couplings of $SU(3)$ and $SU(2)$ coincide at $M_X = 10^{16.2}\text{GeV}$.

Using (3.9), we have the following relation between M_X and M_{st} [11],

$$\log\frac{M_X}{M_{\text{st}}} = \frac{1}{8}\Delta b'\log[(T + \bar{T})|\eta(T)|^4], \quad (3.11)$$

where $\Delta b' \equiv b'_3 - b'_2$. It is remarkable that the value $\log[(T + \bar{T})|\eta(T)|^4]$ is always negative for any value of T . Therefore we need $\Delta b' > 0$ in order to derive $M_X < M_{\text{st}}$ from (3.11). For example we use $M_X = 10^{16.2}\text{GeV}$ and $M_{\text{st}} = 3.73 \times 10^{17}\text{GeV}$ to estimate the value of T in the case with $\Delta b' = 3$. In this case we have $T = 11$.

When the SUSY breaks, the moduli field T could take the non-zero vacuum expectation value. In the SUSY-breaking scenario due to a gaugino condensation, the value of T has been estimated as $T \sim 1.2$ [7]. Further refs.[8] take into account a one-loop effective potential to obtain $T \sim 8$. Thus it seems that the value of T is of order one. Therefore we consider mainly the case where $T < 11$ and $\Delta b' > 3$.

4. Minimal String Model

In this section we study the possibility that we derive the minimal string models consistent with the measured gauge coupling values from the $Z_N \times Z_M$ orbifold construction. We assign the allowed modular weights to the MSSM matter fields and then investigate whether the combinations of the modular weights lead to the suitable threshold corrections explaining the measured coupling constants.

Using (3.9), we have the following relation between $\alpha_{\text{em}}^{-1} \equiv k_1\alpha_1^{-1} + \alpha_2^{-1}$ and α_2^{-1} ,

$$\begin{aligned} \alpha_2^{-1}(\mu) &= \frac{1}{k_1+1}\alpha_{\text{em}}^{-1}(\mu) + \frac{1}{4\pi}\left(1 - \frac{12}{k_1+1}\right)\log\frac{M_{\text{st}}^2}{\mu^2} \\ &\quad - \frac{1}{4\pi}\left(b'_2 - \frac{B'}{k_1+1}\log\right)[(T + \bar{T})|\eta(T)|^4], \end{aligned} \quad (4.1)$$

where $B' \equiv b'_1 + b'_2$. From (4.1) and (3.11), we derive the following equation of $\sin^2 \theta_W \equiv \alpha_{\text{em}}/\alpha_2$,

$$\begin{aligned} \sin^2 \theta_W(\mu) &= \frac{1}{k_1+1} + \frac{\alpha_{\text{em}}(\mu)}{4\pi} \left(\left(1 - \frac{12}{k_1+1}\right) \log \frac{M_{\text{string}}^2}{\mu^2} \right. \\ &\quad \left. - \frac{4}{\Delta b'} (b'_2 - \frac{B'}{k_1+1}) \log \frac{M_X^2}{M_{\text{string}}^2} \right). \end{aligned} \quad (4.2)$$

Then we obtain the following equation,

$$k_1 = \frac{12\Delta b' \log(M_{\text{string}}^2/\mu^2) - 4B' \log(M_X^2/M_{\text{string}}^2) - 4\pi\Delta b' \alpha_{\text{em}}^{-1}(\mu)}{\Delta b' \log(M_{\text{string}}^2/\mu^2) - 4b'_2 \log(M_X^2/M_{\text{string}}^2) - 4\pi\Delta b' \alpha_{\text{em}}^{-1}(\mu) \sin^2 \theta_W(\mu)} - 1, \quad (4.3)$$

where we obtain the unification scale M_X from (3.10) for the case of any SUSY-breaking scale.

At first we assign the allowed modular weights to all the MSSM matter fields, i.e., Q, U, D, L, E and H . We calculate the duality anomaly coefficients of their combinations and restrict ourselves to the case with $\Delta b' > 3$. Then for each combination we estimate the value of k_1 , using (4.3) and (3.10). Here we have to investigate whether or not each combination includes modular weights allowed by this level k_1 through Table 2 and Table 3.

We estimate the allowed values of k_1 in the cases where the SUSY breaks at M_Z and 1TeV. In the third column of Table 1, M_S represents the SUSY-breaking scale. In the case with the SUSY-breaking at 1TeV, we use $\alpha_{\text{em}}^{-1}(1 \text{ TeV}) = 127.2 \pm 0.1$, $\sin^2 \theta_W(1 \text{ TeV}) = 0.2432 \pm 0.0021$ and $\alpha_3^{-1}(1 \text{ TeV}) = 11.48 \pm 0.27$, which are calculated from the values at M_Z through the renormalization group equation in the non-supersymmetric standard model, i.e., $b_3 = -7$, $b_2 = -19/6$ and $b_1 = 41/6$. The results are found in Table 1, whose fourth column shows the maximum values of $\Delta b'$ and the corresponding values of T in the allowed models and fifth column shows the allowed ranges of k_1 ¹. We cannot derive consistent levels from the $Z_3 \times Z_3$ orbifold models in the case where $\Delta b' > 3$ and the SUSY breaks at M_Z . We need at most $\Delta b' = 2$ to derive the measured gauge coupling constants. The range of k_1 in the parenthesis of the fifth column represents the corresponding values of k_1 in the case with $\Delta b' = 2$.

The $Z_2 \times Z_2$ orbifold models are ruled out, because they always derive $\Delta b' < 0$. At this stage, all the orbifold models except the $Z_2 \times Z_2$ are promising for the

¹ The ranges of k_1 obtained in ref.[12] include a minor mistake, and should be replaced by the results shown in Table 1.

minimal string model consistent with the measured gauge coupling constants. We find the range of k_1 as $1 \lesssim k_1 \lesssim 2$. Under these values of k_1 , some of oscillated states are ruled out. The above estimation of the levels includes at most 20% experimental error. Some of the models are possible to have $k_1 = 5/3$, which is the level predicted by GUTs. It is remarkable that the minimal string models from the $Z_N \times Z_M$ orbifold constructions can obtain smaller value of T than ones from the Z_N orbifolds. The latter has the duality anomaly cancellation condition, which constrains the massless spectra.

5. Yukawa Coupling

It is an important problem to derive the realistic Yukawa couplings in addition to the gauge coupling constants at M_Z . The orbifold models have selection rules for the Yukawa couplings [16-19]. A point group selection rule requires that a product of point group elements should be an identity. Moreover, allowed couplings should conserve the $SO(10)$ momenta of the right-moving bosonized fermionic string. Sectors allowed to couple are shown explicitly in ref.[18]. Further the Z_N invariance requires a product of the Z_N phases from the oscillated states to be zero for each plane. Thus the Yukawa couplings are very restricted in the orbifold models, especially for the oscillated states. Actually the minimal string models derived from the Z_N orbifold models in ref.[14] allow at most the Yukawa couplings for the top and bottom quarks as renormalizable couplings. Also, some Yukawa couplings at the low energy could be obtained by nonrenormalizable couplings. However they are suppressed by $1/M_{\text{pl}}$. Therefore we should explain the large values of the Yukawa couplings by the renormalizable couplings. Following ref.[20], we assume that the Yukawa couplings for the second and third generations are due to the renormalizable couplings. Then we search the minimal string models to allow these types of couplings among the combinations of the modular weights obtained at the previous stage.

For example we study the Yukawa couplings of the minimal string models derived from the $Z_2 \times Z_4$ orbifold construction. The point group selection rule and the $SO(10)$ momentum conservation forgive the following couplings [18],

$$\begin{aligned}
U_1 U_2 U_3, \quad U_1 T_{01} T_{03}, \quad U_1 T_{02} T_{02}, \quad U_1 T_{10} T_{10}, \\
U_3 T_{12} T_{12}, \quad T_{02} T_{10} T_{12},
\end{aligned} \tag{5.1}$$

where U_i denotes the untwisted sector associated with the i -th plane. Eq.(5.1) corresponds to the couplings where all of the states have $n = -1$. Also the $Z_2 \times Z_4$ orbifold models allow the following couplings,

$$T_{01}T_{11}T_{12}, \quad T_{03}T_{10}T_{11}, \quad T_{11}T_{11}T_{02}. \quad (5.2)$$

The first and second types of (5.2) correspond to the couplings of states with $n = -1, -1$ and -2 , while the last corresponds to the coupling of states with $n = -1, -2$ and -2 .

Further we have to consider the couplings of the oscillated states. The $Z_2 \times Z_4$ orbifold models do not allow the quark doublets to have the non-zero oscillator number, although other matter fields can be obtained from the oscillated states. For each plane, a product of the Z_N phases due to the oscillators should vanish in order to allow the couplings. We study this selection rule in the $Z_2 \times Z_4$ orbifold models to find that the quark singlets and the Higgs fields in the oscillated states are not allowed to couple with the quark doublets. Similarly the leptons in the oscillated states are impossible to couple with the Higgs fields which have the vanishing oscillator number. Namely any oscillated state does not have couplings. As the results, the allowed couplings are represented by the modular weights as follows,

$$(n_1, n_2, n_3) = (-1, -1, -1), \quad (-1, -1, -2), \quad (-1, -2, -2). \quad (5.3)$$

Similarly we investigate the combinations of the modular weights which allow the Yukawa couplings for the other orbifold models. Some orbifold models allow the quark doublets as well as other matter fields in the oscillated states. However, the couplings of the oscillated states are impossible in the case with $1 \leq k_1 \leq 2.06$, because the Z_N invariance in general requires the larger oscillation number for the oscillated states to couple and the values of k_1 constrain the larger oscillator number of the states. The $Z_2 \times Z_6$ orbifold models have the same combinations of the modular weights to couple as the $Z_2 \times Z_4$, i.e., (5.3). The $Z_4 \times Z_4$, $Z_3 \times Z_6$ and $Z_6 \times Z_6$ orbifold models allow the type of the couplings with $(n_1, n_2, n_3) = (-2, -2, -2)$ in addition to (5.3). The $Z_3 \times Z_3$ orbifold models have the couplings as,

$$(n_1, n_2, n_3) = (-1, -1, -1), \quad (-1, -1, -2), \quad (-2, -2, -2), \quad (5.4)$$

while the $Z_2 \times Z'_6$ orbifold models allow the following couplings,

$$(n_1, n_2, n_3) = (-1, -1, -1), \quad (-1, -2, -2), \quad (-2, -2, -2). \quad (5.5)$$

Note that the $Z_6 \times Z_6$ have the same constraints on the modular weights as the $Z_3 \times Z_6$, and both allow the same types of the Yukawa couplings.

Here we study the minimal string models to allow the Yukawa couplings. For example we study the minimal string models derived from the $Z_2 \times Z_4$ orbifold construction in the case with the SUSY-breaking at M_Z and $\Delta b' > 3$. At the previous stage, these models allow $1.00 \leq k_1 \leq 1.60$ and $\Delta b' \leq 12$, which are shown in the third column of Table 4 as well as Table 1. If we require the top Yukawa coupling as the renormalizable coupling, the minimal string models are constrained and we obtain $1.00 \leq k_1 \leq 1.58$ and $\Delta b' \leq 10$, which are shown in the fourth column of Table 4. Further the Yukawa couplings for the bottom quark as well as the top are allowed in the models with $1.03 \leq k_1 \leq 1.54$ and $\Delta b' \leq 9$, which are shown in the fifth column of Table 4. Similarly the sixth column of the table shows the models to allow the Yukawa couplings for the charm quark as well as the third generation of the quarks, and the seventh column lists the models with the Yukawa couplings for the second and third generations of the quarks. Moreover, the eighth and ninth columns show the models to forgive the Yukawa couplings for the one and two leptons, respectively, in addition to the Yukawa couplings for the two generations of the quarks. The lepton couplings are impossible for the minimal string models derived from the $Z_2 \times Z_4$ orbifold models in the case where the SUSY breaks at M_Z and $\Delta b' > 3$.

Similarly we obtain the minimal string models to allow the Yukawa couplings from the other orbifold models in the case with the SUSY-breaking at M_Z and $\Delta b' > 3$. The results are shown in Table 4. Further Table 5 shows the results in the case with SUSY-breaking at 1TeV and $\Delta b' > 3$. These final results are also shown in the sixth and seventh columns of Table 1. If we forgive the case with $0 < \Delta b' \leq 3$, we find the minimal string models to allow the Yukawa couplings for the two generations of the quarks and the leptons among the $Z_2 \times Z_4$ and $Z_4 \times Z_4$ orbifold models. The Yukawa couplings for the two generations are allowed in the $Z_2 \times Z_4$ orbifold models with $\Delta b' = 1$ and the SUSY-breaking at 1TeV. The range of k_1 for this case is shown in the parenthesis of the seventh column of Table 1. The Yukawa couplings for the two generations are possible in the $Z_4 \times Z_4$ orbifold models with at most $\Delta b' = 2$. For $\Delta b' = 2$, the corresponding values of k_1 are shown in the parentheses of the seventh column. In the case with the SUSY-breaking at M_Z there are four types of the combinations in the $Z_4 \times Z_4$ orbifold models. In all the types, every modular weight of E is equal to -2 , and

four modular weights among L and H are assigned as $n = -2$ and the other has $n = -3$. The modular weights for the other matter fields are assigned as follows,

$$\begin{aligned} & -1, -1, -2 \text{ for } Q, \quad -1, -2, -2 \text{ for } U, \quad 0, -1, -1 \text{ for } D \text{ in Type 1,} \\ & -1, -2, -2 \text{ for } Q, \quad -2, -2, -2 \text{ for } U, \quad 0, -1, -1 \text{ for } D \text{ in Type 2,} \\ & -2, -2, -2 \text{ for } Q, \quad -2, -2, -2 \text{ for } U, \quad -1, -1, -1 \text{ for } D \text{ in Type 3,} \\ & -2, -2, -2 \text{ for } Q, \quad -2, -2, -2 \text{ for } U, \quad 0, -1, -2 \text{ for } D \text{ in Type 4.} \end{aligned}$$

It is remarkable that Type 3 includes only non-oscillated states in the quark sector.

We find that the $Z_2 \times Z'_6$, $Z_2 \times Z_6$, $Z_3 \times Z_6$ and $Z_6 \times Z_6$ orbifold models are promising for the minimal string model to derive the realistic Yukawa couplings. These models derive $1.00 \leq k_1 \leq 1.51$, $\Delta b' \leq 8$ and $T \geq 5.3$. The values of k_1 shown in the last columns of Table 4 and 5 include the 15%, 10% and 13% experimental errors for the $Z_2 \times Z'_6$, $Z_2 \times Z_6$ and $Z_3 \times Z_6$ orbifold models, respectively. The values of k_1 and $\Delta b'$ as well as the combinations of the modular weights are constrained much more than the cases without the restrictions on the Yukawa couplings. The case with $k_1 = 5/3$ is ruled out for any orbifold model.

To show explicitly the obtained models is very useful for model building. In the case of the $Z_2 \times Z'_6$ ($Z_2 \times Z_6$) orbifold models with $\Delta b' > 3$, Table 6 (7) shows the combinations of the modular weights for the minimal string models which derive the measured gauge coupling constants with the SUSY-breaking at M_Z and allow the Yukawa couplings for the two generations of the quarks and the leptons. Note that in Table 6 the combinations # 2, 4 and 5 do not include the oscillated states in the quark sector. Similarly Table 8-1 shows the combinations of the modular weights for the minimal string models derived from the $Z_3 \times Z_6$ and $Z_6 \times Z_6$ orbifold models with $\Delta b' \geq 6$ and the SUSY-breaking at M_Z . It is found in the table that we can obtain the allowed combinations of the modular weights with the values of $\Delta b'$ decreasing by one from the allowed combinations with the larger values of $\Delta b'$ through the following recipe of the substitutions,

$$\begin{aligned} (1) \quad & n = -2 \rightarrow n = -1 \quad \text{in } Q, \\ (2) \quad & n = 0 \rightarrow n = -1 \quad \text{in } U, \\ (3) \quad & n = -1 \rightarrow n = -2 \quad \text{in } U, \end{aligned}$$

where (1) is possible except the combinations with $n = -1$ for all of the three lepton singlets. For example the model of #2, 7 and 10 in Table 8-1 are obtained

from #1 with $\Delta b' = 8$ through the substitutions (1), (2) and (3), respectively. The minimal string models with $\Delta b' = 5$ and 4 are shown in Table 8-2 and Table 8-3, where we omit the combinations obtained from the models with $\Delta b' = 6$ through the above substitutions. In Table 8-2 (8-3), the 32 (69) combinations with $\Delta b' = 5$ (4) are omitted. In Table 8-2, the combination # 6 with $\Delta b' = 5$ corresponds to the model where all of the quarks have the vanishing oscillator number. We obtain the minimal string model from #6, through the substitution (3), although the obtained model with $\Delta b' = 4$ is omitted in Table 8-3. In this model all of the quarks belong to the non-oscillated states.

At last we comment on the mass hierarchy of the quarks and leptons. In the orbifold models, renormalizable couplings of the twisted states are often suppressed by a contribution due to a world-sheet instanton as e^{-aT} , where a depends on a distance between fixed points of the states to couple [16, 19]. This property could explain the mass hierarchy [29, 20]. For example the $Z_2 \times Z'_6$ orbifold models allow the following couplings [18],

$$U_1 U_2 U_3, \quad U_1 T_{13} Y_{13}, \quad U_2 T_{10} Y_{10}, \quad U_3 T_{03} Y_{03}, \quad T_{03} T_{10} Y_{13}, \quad (5.6)$$

as well as

$$T_{01} T_{02} Y_{03}, \quad T_{02} T_{10} Y_{14}, \quad T_{02} T_{11} Y_{13}, \quad T_{01} T_{11} Y_{14}, \quad T_{02} T_{02} Y_{02}. \quad (5.7)$$

Eq.(5.6) corresponds to the couplings of the states with $(n_1, n_2, n_3) = (-1, -1, -1)$ and these couplings do not have the suppression factor due to the world-sheet instanton. Therefore these couplings are not available for the Yukawa couplings for the second generation, although the other couplings (5.7) could be suppressed and possible to explain the suppressed Yukawa couplings for the second generation. Thus in realistic models the Yukawa couplings for the second generation have to belong to the types of (5.7). That might give another constraint on the minimal string model. However we can find the couplings of (5.7) for the second generation in any combination obtained in Table 6. Similarly the models obtained from the $Z_2 \times Z_6$, $Z_3 \times Z_6$ and $Z_6 \times Z_6$ orbifold constructions could always derive the suppressed Yukawa couplings for the second generation. It is very interesting to study assignments of fixed points to the MSSM matter fields to explain the mass hierarchy among the minimal string models obtained the above, as discussed in ref.[20]. That will be investigated elsewhere.

6. Conclusions and Discussions

We have studied the minimal string models which have the gauge coupling constants consistent with the measurements at M_Z . The Yukawa couplings allowed in these models are also investigated. The $Z_2 \times Z'_6$, $Z_2 \times Z_6$, $Z_3 \times Z_6$ and $Z_6 \times Z_6$ orbifold models are promising for the minimal string models with the Yukawa couplings of the two generations as the renormalizable couplings. These models derive $1.00 \leq k_1 \leq 1.51$, $\Delta b' \leq 8$ and $T \leq 5.3$. Tables 6, 7, 8-1, 8-2 and 8-3 show the combinations of the modular weights for the allowed models, explicitly. That is useful for model building. Especially it is very interesting to study which assignments of the MSSM matter fields to the fixed points derive the realistic mass hierarchy of the quarks and the leptons among the models shown in the tables.

In the above analysis, we have assumed that all the soft masses are equal to M_Z or 1TeV. However the string theories in general derive the non-universal soft SUSY-breaking terms [30]. Ref.[31] shows that the unification scale M_X of the SU(3) and SU(2) gauge coupling constants depends on the non-universality of the soft SUSY-breaking terms. The unification scale of the non-universal case is often higher than one of the universal case. In some cases with the non-universal soft masses, the values of $\Delta b' = 2$ or 1 could lead to T of order one. Thus it is important to study the minimal string model taking into account the non-universality. Further it is interesting to extend to the case where T_i have different vacuum expectation values one another. In a similar way we can investigate the extended MSSM with extra matter fields like ref.[32].

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Table 1. Restricted values of k_1 and T

Orbifold	v_1 v_2	M_S	Gauge		Yukawa	
			$\Delta b' (T)$	k_1	$\Delta b' (T)$	k_1
$Z_2 \times Z_2$	(1,0,1)/2	M_Z	—	—	—	—
	(0,1,1)/2	1TeV	—	—	—	—
$Z_3 \times Z_3$	(1,0,2)/3	M_Z	2 (15)	(1.16-1.53)	—	—
	(0,1,2)/3	1TeV	7 (5.8)	1.07-1.35	—	—
$Z_2 \times Z_4$	(1,0,1)/2	M_Z	12 (4.0)	1.00-1.60	—	—
	(0,1,3)/4	1TeV	12 (4.0)	1.00-1.62	1 (28)	(1.00–1.49)
$Z_4 \times Z_4$	(1,0,3)/4	M_Z	12 (4.0)	1.00-1.60	2 (15)	(1.00-1.51)
	(0,1,3)/4	1TeV	12 (4.0)	1.00-1.62	2 (15)	(1.12–1.51)
$Z_2 \times Z_6'$	(1,0,1)/2	M_Z	12 (4.0)	1.00-1.92	6 (6.4)	1.00–1.39
	(1,1,4)/6	1TeV	12 (4.0)	1.00-1.91	6 (6.4)	1.00–1.42
$Z_2 \times Z_6$	(1,0,1)/2	M_Z	18 (3.1)	1.00-1.71	7 (5.8)	1.08–1.31
	(0,1,5)/6	1TeV	18 (3.1)	1.00-1.71	8 (5.3)	1.02–1.34
$Z_3 \times Z_6$	(1,0,2)/3	M_Z	18 (3.1)	1.00-2.06	8 (5.3)	1.01–1.45
	(0,1,5)/6	1TeV	18 (3.1)	1.00-2.04	8 (5.3)	1.02–1.51
$Z_6 \times Z_6$	(1,0,5)/6 (0,1,5)/6	same as $Z_3 \times Z_6$				

 Table 2. Lower-bound of k_1 in twisted sectors (I)

T-sec.				Lower-bound of k_1				
$Z_2 \times Z_2$	$Z_2 \times Z_4$	$Z_4 \times Z_4$	n	Q	D	U	L, H	E
-	T_{01}, T_{03}	$T_{01}, T_{03},$	-1	4/33	16/69	64/69	4/9	16/13
		$T_{10}, T_{13},$	-2,0	-	16/33	64/33	4/5	16/9
		T_{30}, T_{31}	-3,1	-	-	-	4	16/5
			-4,2	-	-	-	-	16
$T_{01}, T_{10},$ T_{11}	$T_{02}, T_{10},$ T_{12}	$T_{02}, T_{20},$	-1	1/6	4/15	16/15	1/2	4/3
		T_{22}	-2,0	-	-	-	-	4
-	T_{11}	$T_{11}, T_{12},$	-2	4/15	16/51	64/51	4/7	16/11
		T_{21}	-3	-	16/15	64/15	4/3	16/7
			-1, -4	-	-	-	-	16/3

Table 3. Lower-bound of k_1 in twisted sectors (II)

T-Sec.					Lower-bound of k_1					
$Z_3 \times Z_3$	$Z_2 \times Z_6'$	$Z_2 \times Z_6$	$Z_3 \times Z_6$	$Z_6 \times Z_6$	n	Q	D	U	L, H	E
-	-	T_{01}, T_{05}	T_{01}, T_{05}	$T_{01}, T_{05},$ $T_{10}, T_{15},$ T_{50}, T_{51}	-1	1/10	4/19	16/19	9/22	36/31
					-2,0	1/4	4/13	16/13	9/16	36/25
					-3,1	-	4/7	16/7	9/10	36/19
					-4,2	-	4	16	9/4	36/13
					-5,3	-	-	-	-	36/7
					-6,4	-	-	-	-	36
$T_{01}, T_{02},$ $T_{10}, T_{12},$ T_{20}, T_{21}	-	T_{02}, T_{04}	$T_{02}, T_{04},$ $T_{10}, T_{14},$ T_{20}, T_{22}	$T_{02}, T_{04},$ $T_{20}, T_{24},$ T_{40}, T_{42}	-1	1/7	1/4	1	9/19	9/7
					-2,0	-	1	4	9/7	9/4
					-3,1	-	-	-	-	9
-	$T_{03}, T_{10},$ T_{13}	$T_{03}, T_{10},$ T_{13}	T_{03}	$T_{03}, T_{30},$ T_{33}	-1	1/6	4/15	16/15	1/2	4/3
					-2,0	-	-	-	-	4
-	-	$T_{11}, T_{12},$	T_{11}, T_{13}	$T_{12}, T_{13},$ $T_{21}, T_{23},$ T_{31}, T_{32}	-2	1/4	4/13	16/13	9/16	36/25
					-3	-	4/7	16/7	9/10	36/19
					-4	-	4	16	9/4	36/13
					-1,-5	-	-	-	-	36/7
					-6	-	-	-	-	36
T_{11}	T_{02}	-	T_{12}	T_{22}	-2	1/3	1/3	4/3	3/5	3/2
					-3	-	-	-	3	3
-	$T_{01}, T_{11},$ T_{14}	-	T_{21}	$T_{11}, T_{14},$ T_{41}	-2	1/6	4/15	16/15	1/2	4/3
					-3	-	4/9	16/9	3/4	12/7
					-1,-4	-	4/3	16/3	3/2	12/5
					-5	-	-	-	-	4
					-6,0	-	-	-	-	12

Table 4. Values of k_1 and $\Delta b'$ with SUSY-breaking at M_Z

Orbifold		Coupling						
			t	b	c	s	τ	μ
$Z_2 \times Z_4$	$\Delta b'$	12 (4.0)	10 (4.5)	9 (4.9)	9 (4.9)	7 (5.8)	—	—
	k_1	1.00-1.60	1.00-1.58	1.03-1.54	1.03-1.54	1.03-1.51	—	—
$Z_4 \times Z_4$	$\Delta b'$	12 (4.0)	11 (4.2)	9 (4.9)	9 (4.9)	7 (5.8)	—	—
	k_1	1.00-1.60	1.00-1.58	1.00-1.54	1.03-1.54	1.03-1.51	—	—
$Z_2 \times Z'_6$	$\Delta b'$	12 (4.0)	10 (4.5)	9 (4.9)	8 (5.3)	8 (5.3)	7 (5.8)	6 (6.5)
	k_1	1.00-1.92	1.00-1.85	1.00-1.59	1.00-1.59	1.00-1.59	1.00-1.53	1.00-1.39
$Z_2 \times Z_6$	$\Delta b'$	18 (3.1)	16 (3.3)	13 (3.8)	12 (4.0)	10 (4.5)	9 (4.9)	7 (5.8)
	k_1	1.00-1.71	1.00-1.65	1.00-1.59	1.00-1.59	1.00-1.54	1.01-1.35	1.08-1.31
$Z_3 \times Z_6$ ($Z_6 \times Z_6$)	$\Delta b'$	18 (3.1)	16 (3.3)	13 (3.8)	12 (4.0)	10 (4.5)	9 (4.9)	8 (5.3)
	k_1	1.00-2.06	1.00-1.94	1.00-1.91	1.00-1.82	1.00-1.82	1.01-1.58	1.01-1.45

Table 5. Values of k_1 and $\Delta b'$ with SUSY-breaking at 1TeV

Orbifold		Coupling						
			t	b	c	s	τ	μ
$Z_3 \times Z_3$	$\Delta b'$	7 (5.8)	6 (6.5)	—	—	—	—	—
	k_1	1.07-1.35	1.07-1.35	—	—	—	—	—
$Z_2 \times Z_4$	$\Delta b'$	12 (4.0)	11 (4.2)	9 (4.9)	9 (4.9)	8 (5.3)	7 (5.8)	—
	k_1	1.00-1.62	1.00-1.60	1.03-1.56	1.03-1.56	1.03-1.52	1.09-1.36	—
$Z_4 \times Z_4$	$\Delta b'$	12 (4.0)	11 (4.2)	9 (4.9)	9 (4.9)	8 (5.3)	7 (5.8)	—
	k_1	1.00-1.62	1.00-1.60	1.03-1.56	1.03-1.56	1.03-1.53	1.03-1.49	—
$Z_2 \times Z'_6$	$\Delta b'$	12 (4.0)	10 (4.5)	9 (4.9)	8 (5.3)	8 (5.3)	7 (5.8)	6 (6.5)
	k_1	1.00-1.91	1.00-1.85	1.00-1.72	1.00-1.72	1.00-1.72	1.00-1.55	1.00-1.42
$Z_2 \times Z_6$	$\Delta b'$	18 (3.1)	16 (3.3)	13 (3.8)	12 (4.0)	10 (4.5)	9 (4.9)	8 (5.3)
	k_1	1.00-1.71	1.00-1.66	1.00-1.61	1.00-1.61	1.00-1.56	1.01-1.46	1.02-1.34
$Z_3 \times Z_6$ ($Z_6 \times Z_6$)	$\Delta b'$	18 (3.1)	16 (3.3)	13 (3.8)	12 (4.0)	10 (4.5)	9 (4.9)	8 (5.3)
	k_1	1.00-2.04	1.00-1.93	1.00-1.91	1.00-1.82	1.00-1.82	1.01-1.60	1.02-1.51

Table 6. Minimal String Models from $Z_2 \times Z'_6$ orbifold

#	$\Delta b'^k$	Q	U	D	L, H	E
1	6	-2, -2, -3	-1, -1, -2	-1, -1, -1	-2, -2, -2-, 2, -3	-2, -2, -2
2	5	-2, -2, -2	-1, -1, -1	-1, -1, -1	-2, -2, -2-, 2, -3	-2, -2, -2
3	5	-2, -2, -3	-1, -1, -2	-1, -1, -1	-2, -2, -2-, 2, -3	-2, -2, -2
4	4	-1, -2, -2	-1, -1, -1	-1, -1, -1	-2, -2, -2-, 2, -3	-2, -2, -2
5	4	-2, -2, -2	-1, -1, -2	-1, -1, -1	-2, -2, -2-, 2, -3	-2, -2, -2
6	4	-2, -2, -3	-1, -1, -2	-1, -1, -2	-2, -2, -2-, 2, -3	-2, -2, -2
7	4	-2, -2, -3	-1, -2, -2	-1, -1, -1	-2, -2, -2-, 2, -3	-1, -2, -2
8	4	-2, -2, -3	-1, -2, -2	-1, -1, -1	-2, -2, -2-, 2, -3	-2, -2, -2

Table 7. Minimal String Models from $Z_2 \times Z_6$ orbifold

#	$\Delta b'^k$	Q	U	D	L, H	E
1	7	-2, -2, -2	-1, -1, -1	1, -1, -1	-2, -2, -2-, 2, -3	-1, -1, -1
2	6	-2, -2, -2	-1, -1, -2	1, -1, -1	-2, -2, -2-, 2, -3	-1, -1, -1
3	5	-1, -2, -2	-1, -1, -2	1, -1, -1	-2, -2, -2-, 2, -3	-1, -1, -1
4	4	-1, -2, -2	-1, -2, -2	1, -1, -1	-2, -2, -2-, 2, -3	-1, -1, -1

Table 8-1. Minimal String Models from $Z_3 \times Z_6$ and $Z_6 \times Z_6$ orbifolds with $\Delta b' \geq 6$

#	$\Delta b'^k$	Q	U	D	L, H	E
1	8	-2, -2, -2	0, -1, -1	1, -1, -1	-2, -2, -2-, 2, -3	-2, -2, -2
2	7	-1, -2, -2	0, -1, -1	1, -1, -1	-2, -2, -2-, 2, -3	-2, -2, -2
3	7	-2, -2, -2	0, -1, -2	0, -1, -1	-2, -2, -2-, 2, -3	-2, -2, -2
4	7	-2, -2, -2	0, -1, -1	1, -1, -2	-2, -2, -2-, 2, -3	-2, -2, -2
5	7	-2, -2, -2	-1, -1, -1	1, -1, -1	-2, -2, -2-, 2, -3	-1, -1, -1
6	7	-2, -2, -2	-1, -1, -1	1, -1, -1	-2, -2, -2-, 2, -3	-1, -2, -2
7	7	-2, -2, -2	-1, -1, -1	1, -1, -1	-2, -2, -2-, 2, -3	-2, -2, -2
8	7	-2, -2, -2	0, -1, -2	1, -1, -1	-2, -2, -2-, 2, -3	-1, -1, -1
9	7	-2, -2, -2	0, -1, -2	1, -1, -1	-2, -2, -2-, 2, -3	-1, -2, -2
10	7	-2, -2, -2	0, -1, -2	1, -1, -1	-2, -2, -2-, 2, -3	-2, -2, -2
11	6	-1, -1, -2	0, -1, -1	1, -1, -1	-2, -2, -2-, 2, -3	-2, -2, -2
12	6	-1, -2, -2	0, -1, -1	0, -1, -1	-2, -2, -2-, 2, -3	-2, -2, -2
13	6	-1, -2, -2	0, -1, -1	1, -1, -2	-2, -2, -2-, 2, -3	-2, -2, -2
14	6	-1, -2, -2	-1, -1, -1	1, -1, -1	-2, -2, -2-, 2, -3	-1, -2, -2
15	6	-1, -2, -2	-1, -1, -1	1, -1, -1	-2, -2, -2-, 2, -3	-2, -2, -2
16	6	-1, -2, -2	0, -1, -2	1, -1, -1	-2, -2, -2-, 2, -3	-1, -2, -2
17	6	-1, -2, -2	0, -1, -2	1, -1, -1	-2, -2, -2-, 2, -3	-2, -2, -2
18	6	0, -2, -2	0, -1, -1	1, -1, -1	-2, -2, -2-, 2, -3	-2, -2, -2
19	6	-2, -2, -2	-1, -1, -1	1, -1, -1	-1, -2, -2-, 2, -3	-2, -2, -2
20	6	-2, -2, -2	-1, -1, -1	1, -1, -1	-2, -2, -2-, 2, -2	-2, -2, -2
21	6	-2, -2, -2	-1, -1, -1	0, -1, -1	-2, -2, -2-, 2, -3	-2, -2, -2
22	6	-2, -2, -2	-1, -1, -1	1, -1, -2	-2, -2, -2-, 2, -3	-2, -2, -2
23	6	-2, -2, -2	-1, -1, -2	1, -1, -1	-2, -2, -2-, 2, -3	-1, -1, -1
24	6	-2, -2, -2	-1, -1, -2	1, -1, -1	-2, -2, -2-, 2, -3	-1, -2, -2
25	6	-2, -2, -2	-1, -1, -2	1, -1, -1	-2, -2, -2-, 2, -3	-2, -2, -2
26	6	-2, -2, -2	0, -1, -2	1, -1, -1	-1, -2, -2-, 2, -3	-2, -2, -2
27	6	-2, -2, -2	0, -1, -2	1, -1, -1	-2, -2, -2-, 2, -2	-2, -2, -2
28	6	-2, -2, -2	0, -1, -2	0, -1, -1	-2, -2, -2-, 2, -3	-2, -2, -2
29	6	-2, -2, -2	0, -1, -2	1, -1, -2	-2, -2, -2-, 2, -3	-2, -2, -2
30	6	-2, -2, -2	0, -2, -2	1, -1, -1	-2, -2, -2-, 2, -3	-1, -1, -1
31	6	-2, -2, -2	0, -2, -2	1, -1, -1	-2, -2, -2-, 2, -3	-1, -2, -2
32	6	-2, -2, -2	0, -2, -2	1, -1, -1	-2, -2, -2-, 2, -3	-2, -2, -2

Table 8-2. Minimal String Models from $Z_3 \times Z_6$ and $Z_6 \times Z_6$ orbifolds with $\Delta b' = 5$

#	$\Delta b'^k$	Q	U	D	L, H	E
1	5	-1, -2, -2	-1, -1, -2	1, -1, -1	-2, -2, -2-, 2, -3	-1, -1, -1
2	5	-1, -2, -2	0, -2, -2	1, -1, -1	-2, -2, -2-, 2, -3	-1, -1, -1
3	5	0, -2, -2	0, -1, -1	0, -1, -1	-2, -2, -2-, 2, -3	-2, -2, -2
4	5	0, -2, -2	0, -1, -1	1, -1, -2	-2, -2, -2-, 2, -3	-2, -2, -2
5	5	0, -2, -2	-1, -1, -1	1, -1, -1	-2, -2, -2-, 2, -3	-1, -2, -2
6	5	-2, -2, -2	-1, -1, -1	-1, -1, -1	-2, -2, -2-, 2, -3	-2, -2, -2
7	5	-2, -2, -2	-1, -1, -1	0, -1, -2	-2, -2, -2-, 2, -3	-2, -2, -2
8	5	-2, -2, -2	-1, -1, -1	1, -2, -2	-2, -2, -2-, 2, -3	-2, -2, -2
9	5	-2, -2, -2	-1, -1, -2	0, -1, -1	-2, -2, -2-, 2, -3	-1, -2, -2
10	5	-2, -2, -2	-1, -1, -2	1, -1, -2	-2, -2, -2-, 2, -3	-1, -2, -2
11	5	-2, -2, -2	0, -1, -2	-1, -1, -1	-2, -2, -2-, 2, -3	-2, -2, -2
12	5	-2, -2, -2	0, -1, -2	0, -1, -2	-2, -2, -2-, 2, -3	-2, -2, -2
13	5	-2, -2, -2	0, -1, -2	1, -2, -2	-2, -2, -2-, 2, -3	-2, -2, -2
14	5	-2, -2, -2	-1, -2, -2	1, -1, -1	-2, -2, -2-, 2, -3	-1, -1, -2
15	5	-2, -2, -2	0, -2, -2	0, -1, -1	-2, -2, -2-, 2, -3	-1, -2, -2
16	5	-2, -2, -2	0, -2, -2	1, -1, -2	-2, -2, -2-, 2, -3	-1, -2, -2

Table 8-3. Minimal String Models from $Z_3 \times Z_6$ and $Z_6 \times Z_6$ orbifolds with $\Delta b' = 4$

#	$\Delta b'^k$	Q	U	D	L, H	E
1	4	0, -1, -1	0, -1, -1	1, -1, -1	-2, -2, -2-, 2, -3	-1, -2, -2
2	4	0, -1, -2	0, -1, -2	1, -1, -1	-2, -2, -2-, 2, -3	-1, -2, -2
3	4	-1, -2, -2	-1, -2, -2	1, -1, -1	-2, -2, -2-, 2, -3	-1, -1, -1
4	4	-1, -2, -2	-1, -1, -2	1, -1, -1	-2, -2, -2-, 2, -3	-1, -1, -2
5	4	-1, -2, -2	-1, -2, -2	1, -1, -1	-2, -2, -2-, 2, -3	-1, -2, -2
6	4	0, -2, -2	-1, -1, -1	1, -1, -1	-1, -2, -2-, 2, -3	-2, -2, -2
7	4	0, -2, -2	-1, -1, -1	1, -1, -1	-2, -2, -2-, 2, -2	-2, -2, -2
8	4	0, -2, -2	0, -1, -2	1, -1, -1	-1, -2, -2-, 2, -3	-2, -2, -2
9	4	0, -2, -2	0, -1, -2	1, -1, -1	-2, -2, -2-, 2, -2	-2, -2, -2
10	4	0, -2, -2	0, -2, -2	1, -1, -1	-2, -2, -2-, 2, -3	-1, -2, -2
11	4	-2, -2, -2	-1, -1, -2	0, -1, -1	-1, -2, -2-, 2, -3	-2, -2, -2
12	4	-2, -2, -2	-1, -1, -2	0, -1, -1	-2, -2, -2-, 2, -2	-2, -2, -2
13	4	-2, -2, -2	-1, -1, -2	1, -1, -2	-1, -2, -2-, 2, -3	-2, -2, -2
14	4	-2, -2, -2	-1, -2, -2	1, -1, -2	-2, -2, -2-, 2, -2	-2, -2, -2
15	4	-2, -2, -2	-1, -2, -2	1, -1, -1	-2, -2, -2-, 2, -2	-1, -2, -2
16	4	-2, -2, -2	-1, -2, -2	0, -1, -1	-2, -2, -2-, 2, -3	-1, -1, -1
17	4	-2, -2, -2	-1, -2, -2	1, -1, -2	-2, -2, -2-, 2, -3	-1, -1, -1
18	4	-2, -2, -2	0, -2, -2	0, -1, -1	-1, -2, -2-, 2, -3	-2, -2, -2
19	4	-2, -2, -2	0, -2, -2	0, -1, -1	-2, -2, -2-, 2, -2	-2, -2, -2
20	4	-2, -2, -2	0, -2, -2	0, -1, -2	-1, -2, -2-, 2, -3	-2, -2, -2
21	4	-2, -2, -2	0, -2, -2	1, -1, -2	-1, -2, -2-, 2, -3	-2, -2, -2